Calculus I — Math 141

Name: _____

Instructions: You must show all work to earn full credit. No calculators allowed. If you do not have room in the given space to answer a question, use the back of the formula sheet and *indicate clearly* which work goes with which problem.

Problem	Maximum Points	Your Points
1	12	
2	19	
2	12	
3	8	
4	12	
5	18	
6	12	
7	12	
Q	6	
0	0	
9	8	
Total	100	

1. (12 points) Find an equation for the tangent line to the graph $y = x^2 - 5$ at the point x = 4.

2. (12 points)

(a) Find
$$\sin\left(\frac{5\pi}{4}\right)$$
.

(b) Find **all** solutions to $\cos(x) = -\frac{\sqrt{2}}{2}$.

3. (8 points) Let $g(x) = x^5 + x^3 + 1$. Without solving the equation g(x) = 0, use a theorem to show that there is at least one number c in the interval [-1, 1] such that g(c) = 0. Be sure you explain why the theorem applies.

4. (12 points) Use the *definition of the derivative* to find f'(x) for $f(x) = \frac{1}{2+x}$.

5. (18 points) Compute the following derivatives.

(a)
$$\frac{d}{dx} \frac{x^2}{\sqrt{x}}$$

(b)
$$\frac{d}{dx}(3x+1)(2x+1)$$

Hint: The answer is not $(3)(2) = 6$.

(c) Find the 2nd derivative
$$\frac{d^2y}{dx^2}$$
 of $y = 5x^3 + 4x^2 + \cos x$

- 6. (12 points) Suppose someone throws a football straight up in the air and the height of the football is $h(t) = 32t 32t^2$ feet where t is the number of seconds since the ball was thrown.
 - (a) What is the velocity of the football as a function of time.

(b) At what time does the ball stop rising and start falling?

7. (12 points) Find exact values for the following limits.

(a)
$$\lim_{x \to 2} \frac{x^2 - 7x + 10}{x^2 - x - 2}$$

(b)
$$\lim_{x \to 1^{-}} \frac{1}{x^2 - 1}$$

8. (6 points) Find functions f(x) and g(x) such that the composition f(g(x)) is $\sqrt{1-x^2}$.



- 9. (8 points) Using the graph of f(x) above, find the following limits. If a limit does not exist, then explain why.
 - (a) What is $\lim_{x\to -2} f(x)$?
 - (b) What is $\lim_{x\to 2} f(x)$?
 - (c) What is the $\lim_{x\to 0} f(x)$?
 - (d) Identify all x-values where f(x) is discontinuous.

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Common Trigonometric Ratios

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\cos \theta$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	1/2	0
$\sin \theta$	0	1/2	$\sqrt{2}/2$	$\sqrt{3}/2$	1

Obscure Trigonometry Ratios

$$\cot \theta = \frac{\cos \theta}{\sin \theta}, \qquad \csc \theta = \frac{1}{\sin \theta}$$

Special Trigonometry Limits

$$\lim_{x \to 0} \frac{\sin x}{x} = 1, \qquad \lim_{x \to 0} \frac{\cos x - 1}{x} = 0$$